

$N = 2$ SUPER- $W_3^{(2)}$ ALGEBRA IN SUPERFIELDS

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Abstract

We present a manifestly $N = 2$ supersymmetric formulation of $N = 2$ super- $W_3^{(2)}$ algebra (its classical version) in terms of the spin 1 unconstrained supercurrent generating a $N = 2$ superconformal subalgebra and the spins 1/2, 2 bosonic and spins 1/2, 2 fermionic constrained supercurrents. We consider a superfield reduction of $N = 2$ super- $W_3^{(2)}$ to $N = 2$ super- W_3 and construct a family of evolution equations for which $N = 2$ super- $W_3^{(2)}$ provides the second hamiltonian structure.

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1 Introduction

In recent years a plenty of various superextensions of nonlinear W algebras were constructed and studied from different points of view, both at the classical and quantum levels (see, e.g., [1] and references therein). An interesting class of bosonic W algebras is so called quasisuperconformal algebras which include, besides the bosonic currents with the canonical integer conformal spins, those with half-integer spins [2, 3, 4]. The simplest example of such an algebra is the Polyakov-Bershadsky $W_3^{(2)}$ algebra [5, 6]. It is a bosonic analog of the linear $N = 2$ superconformal algebra (SCA) [7]: apart from two currents with the spins 2 and 1 (conformal stress-tensor and $U(1)$ Kac-Moody (KM) current), it contains two bosonic currents with spins $3/2$. For the currents to form a closed set (with the relevant Jacobi identities satisfied), the OPE between the spin $3/2$ currents should necessarily include a quadratic nonlinearity in the $U(1)$ KM current. So $W_3^{(2)}$, in contrast to its superconformal prototype, is a nonlinear algebra.

It is natural to seek for supersymmetric extensions of this type of W algebras and to see how they can be formulated in terms of superfields. First explicit example of such an extension, $N = 2$ super- $W_3^{(2)}$ algebra, has been constructed at the classical level in [8] (its quantum version is given in [9]). It involves fermionic currents with integer spins 1 and 2 and contains both $N = 2$ SCA and $W_3^{(2)}$ as subalgebras. Actually, it can be regarded as a nonlinear closure of these two algebras.¹

Curiously enough, the spin content of the currents of $N = 2$ super- $W_3^{(2)}$ algebra is such that they cannot be immediately arranged into $N = 2$ supermultiplets with respect to the $N = 2$ SCA which is manifest in the formulation given in [8]. This means that $N = 2$ super- $W_3^{(2)}$, as it stands, does not admit the standard $N = 2$ superfield description, in contrast, e.g., to $N = 2$ super- W_3 algebra [12, 13]. One can still wonder whether any other superfield formulation exists, perhaps with composite currents involved into the game. Recall that it is very advantageous to have a superfield description because it radically simplifies computations and allows to present all results in an explicitly supersymmetric concise form.

In the present paper we show that $N = 2$ super- $W_3^{(2)}$ algebra of ref. [8] admits a nice superfield description with respect to *another* $N = 2$ superconformal subalgebra which is implicit in the original formulation. An unusual novel feature of this description is that some of the relevant supercurrents are given by $N = 2$ superfields subjected to nonlinear constraints. Using the superfield formulation constructed, we demonstrate that $N = 2$ super- W_3 algebra follows from $N = 2$ super- $W_3^{(2)}$ by a secondary hamiltonian reduction, like W_3 follows from $W_3^{(2)}$ [14, 11]. We also construct a family of $N = 2$ superfield evolution equations with $N = 2$ super- $W_3^{(2)}$ as the second hamiltonian structure.

2 Preliminaries

For the reader's convenience we review here the salient features of $N = 2$ super- $W_3^{(2)}$ algebra in terms of component currents [8].

¹Zamolodchikov's W_3 algebra [10] can be nonlinearly embedded into $W_3^{(2)}$ [11], so it also forms a subalgebra in $N = 2$ $W_3^{(2)}$ (in some special basis for the generating currents of the latter).

A powerful method of constructing conformal (super)algebras is the hamiltonian reduction method [16, 3, 17]. In this approach one writes down a gauge potential \mathcal{A} valued in the appropriate (super)algebra g and then constrain some components of \mathcal{A} to be equal to constants. From the residual gauge transformations of the remaining components of \mathcal{A} one can immediately read off the OPEs of some conformal W (super)algebra, with these components as the generating currents. Since the residual gauge transformations clearly form a closed set, the Jacobi identities of the resulting W algebra prove to be automatically satisfied.

A straightforward application of hamiltonian reduction to superalgebra $sl(3|2)$ gives rise to the classical $N = 2$ super- W_3 algebra [12]. In [8] a different choice of constraints has been made (it corresponds to some non-principal embedding of $sl(2)$ into the bosonic $sl(3) \times sl(2)$ subalgebra of $sl(3|2)$). The residual gauge transformations of the remaining currents yield just the $N = 2$ super- $W_3^{(2)}$ algebra we will deal with here.

More precisely, starting with the following constrained $sl(3|2)$ gauge potential \mathcal{A}

$$\mathcal{A} = \frac{1}{c} \begin{pmatrix} 2J_s - 3J_w & G^+ & T_1 & S_1 & S_2 \\ 0 & 2J_s - 6J_w & G^- & 0 & S \\ 1 & 0 & 2J_s - 3J_w & 0 & S_1 \\ \bar{S}_1 & \bar{S} & \bar{S}_2 & 3J_s - 6J_w & T_2 \\ 0 & 0 & \bar{S}_1 & 1 & 3J_s - 6J_w \end{pmatrix}, \quad (2.1)$$

where $\{J_s, J_w, G^+, G^-, T_1, T_2\}$ and $\{S_1, \bar{S}_1, S, \bar{S}, S_2, \bar{S}_2\}$ are, respectively, bosonic and fermionic currents, one can easily find the residual gauge transformations which preserve this particular form of \mathcal{A} . They correspond to the following parameters

$$l_1, l_3, a_3, a_5, a_6, a_8, b_3, b_5, (b_1 + b_6), c_4, c_5, (c_1 + c_6) \quad (2.2)$$

in the standard infinitesimal gauge transformation of \mathcal{A}

$$\delta\mathcal{A} = \partial\Lambda + [\mathcal{A}, \Lambda], \quad (2.3)$$

with Λ being a $sl(3|2)$ -valued matrix of the parameters

$$\Lambda = \begin{pmatrix} 2l_1 + l_2 + l_3 & a_1 & a_2 & b_1 & b_2 \\ a_3 & 2l_1 - 2l_3 & a_4 & b_3 & b_4 \\ a_5 & a_6 & 2l_1 - l_2 + l_3 & b_5 & b_6 \\ c_1 & c_2 & c_3 & 3l_1 + l_4 & a_7 \\ c_4 & c_5 & c_6 & a_8 & 3l_1 - l_4 \end{pmatrix}. \quad (2.4)$$

The remaining twelve combinations of the parameters are expressed through these ones and the currents. After representing these transformations in the form

$$\begin{aligned} \delta\phi(z_1) &= c \oint dz_2 \left[-6l_1 J_s + 18l_3 J_w + a_3 G^+ + a_6 G^- + 3a_5 T_1 - 3a_8 T_2 + b_3 \bar{S} + b_5 \bar{S}_2 \right. \\ &\quad \left. + (b_1 + b_6) \bar{S}_1 - c_4 S_2 - c_5 S - (c_1 + c_6) S_1 \right] \phi(z_1), \end{aligned} \quad (2.5)$$

where $\phi(z)$ is any current, a self-consistent set of OPEs for the currents can be extracted from eq.(2.5).

To understand why this superalgebra was called $N = 2$ super- $W_3^{(2)}$, it is instructive to redefine the currents in the following way

$$T_w = T_1 - \frac{1}{c} S_1 \bar{S}_1 + \frac{3}{c} J_w^2 \quad , \quad T_s = -T_2 - \frac{1}{c} S_1 \bar{S}_1 + \frac{1}{c} J_s^2. \quad (2.6)$$

Then the currents $\{J_w, G^+, G^-, T_w\}$ and $\{J_s, S, \bar{S}, T_s\}$ can be shown to obey the following OPEs²

$$\begin{aligned} J_w(z_1)J_w(z_2) &= \frac{\frac{c}{6}}{z_{12}^2} \quad , \quad J_w(z_1)T_w(z_2) = \frac{J_w}{z_{12}^2}, \\ J_w(z_1)G^\pm(z_2) &= \mp \frac{1}{2} \frac{G^\pm}{z_{12}} \quad , \quad T_w(z_1)G^\pm(z_2) = \frac{3}{2} \frac{G^\pm}{z_{12}^2} + \frac{G^{\pm'}}{z_{12}}, \\ T_w(z_1)T_w(z_2) &= -\frac{3c}{z_{12}^4} + \frac{2T_w}{z_{12}^2} + \frac{T'_w}{z_{12}}, \\ G^+(z_1)G^-(z_2) &= \frac{2c}{z_{12}^3} - \frac{6J_w}{z_{12}^2} - (T_w - \frac{12}{c} J_w^2 + 3J'_w) \frac{1}{z_{12}}, \end{aligned} \quad (2.7)$$

$$\begin{aligned} J_s(z_1)J_s(z_2) &= \frac{\frac{c}{2}}{z_{12}^2} \quad , \quad J_s(z_1)T_s(z_2) = \frac{J_s}{z_{12}^2}, \\ J_s(z_1)S(z_2) &= \frac{1}{2} \frac{S}{z_{12}} \quad , \quad J_s(z_1)\bar{S}(z_2) = -\frac{1}{2} \frac{\bar{S}}{z_{12}}, \\ S(z_1)\bar{S}(z_2) &= \frac{2c}{z_{12}^3} + \frac{2J_s}{z_{12}^2} + \frac{T_s + J'_s}{z_{12}}, \\ T_s(z_1)S(z_2) &= \frac{3}{2} \frac{S}{z_{12}^2} + \frac{S'}{z_{12}} \quad , \quad T_s(z_1)\bar{S}(z_2) = \frac{3}{2} \frac{\bar{S}}{z_{12}^2} + \frac{\bar{S}'}{z_{12}}, \\ T_s(z_1)T_s(z_2) &= \frac{3c}{z_{12}^4} + \frac{2T_s}{z_{12}^2} + \frac{T'_s}{z_{12}}. \end{aligned} \quad (2.8)$$

So they form $W_3^{(2)}$ and $N = 2$ SCA with the related central charges.

Thus we are eventually left with the set of currents which includes those generating $W_3^{(2)}$ and $N = 2$ SCA, as well as four extra fermionic currents $\{S_1, \bar{S}_1, S_2, \bar{S}_2\}$ with the integer spins $\{1, 1, 2, 2\}$. The spin-statistics content of $N = 2$ super- $W_3^{(2)}$ algebra is summarized in Table 1.

Currents	J_s	J_w	S_1	\bar{S}_1	G^+	G^-	S	\bar{S}	T_s	T_w	S_2	\bar{S}_2
Spins	1^B	1^B	1^F	1^F	$\frac{3}{2}^B$	$\frac{3}{2}^B$	$\frac{3}{2}^F$	$\frac{3}{2}^F$	2^B	2^B	2^F	2^F

Table 1.

²Hereafter, we explicitly write down only singular terms in OPEs. All the currents appearing in the right hand sides of the OPEs are evaluated at the point z_2 ($z_{12} = z_1 - z_2$).

All the currents with the aforementioned spins, except for T_s and T_w , are primary with respect to the following Virasoro stress-tensor T having a zero central charge

$$T = T_s + T_w + \frac{4}{c} S_1 \bar{S}_1 - \frac{4}{c} J_s^2 + \frac{12}{c} J_w J_s - \frac{12}{c} J_w^2 \quad . \quad (2.9)$$

The currents T_s and T_w are quasiprimary with the central charges $3c$ and $-3c$, respectively. It can be checked that in this $N = 2$ super- $W_3^{(2)}$ algebra there exists no basis for the currents such that all the currents are primary with respect to some (improved) Virasoro stress-tensor.

The whole set of OPEs of $N = 2$ super- $W_3^{(2)}$ algebra in terms of these currents is given in Appendix.

3 $N = 2$ super- $W_3^{(2)}$ algebra in terms of $N = 2$ supercurrents

Despite the fact that $N = 2$ super- $W_3^{(2)}$ algebra has an equal number of bosonic and fermionic currents, it is unclear how they could be arranged into $N = 2$ supermultiplets. The main obstruction against the existence of a superfield description is the fact that in the superalgebra considered the numbers of currents with integer and half-integer spins do not coincide, while any $N = 2$ superfield clearly contains the equal number of components with integer and half-integer spins.

To find a way to construct the $N = 2$ super- $W_3^{(2)}$ algebra in terms of $N = 2$ superfields, two features of its components OPEs (2.7), (2.8), (A.1) must be taken into account.

First of all, $N = 2$ super- $W_3^{(2)}$ algebra is nonlinear. This means that one may choose the basis for its generating currents in many different ways. The transformations relating different bases must be invertible but in general they are nonlinear and can include derivatives of the currents along with the currents themselves.

Secondly, we would like to stress that the OPEs (2.7), (2.8), (A.1) do not fix the scale of fermionic $S_1, \bar{S}_1, S, \bar{S}, S_2, \bar{S}_2$ and bosonic G^+, G^- currents. Moreover, keeping in mind that all these currents possess definite charges with respect to the J_w and J_s $U(1)$ currents, one can introduce a new “improved” stress-tensor

$$\mathcal{T} = T + bJ'_w + gJ'_s, \quad (3.1)$$

with respect to which the currents $G^+, G^-, S_1, \bar{S}_1, S, \bar{S}, S_2, \bar{S}_2$, still remaining primary, have the dimensions (spins) listed in Table 2.

Currents	G^+	G^-	S_1	\bar{S}_1
Spins	$\frac{3}{2} + \frac{b}{2} + g$	$\frac{3}{2} - \frac{b}{2} - g$	$1 + \frac{b}{6} + \frac{g}{2}$	$1 - \frac{b}{6} - \frac{g}{2}$
Currents	S	\bar{S}	S_2	\bar{S}_2
Spins	$\frac{3}{2} - \frac{b}{3} - \frac{g}{2}$	$\frac{3}{2} + \frac{b}{3} + \frac{g}{2}$	$2 + \frac{b}{6} + \frac{g}{2}$	$2 - \frac{b}{6} - \frac{g}{2}$

Table 2.

Thus we cannot exclude the possibility that in some nonlinear basis the generating currents could have appropriate spins to be organized into supermultiplets with respect to some new $N = 2$ SCA.

Fortunately, just this situation takes place for the superalgebra under consideration.

In order to demonstrate this, let us pass to the new basis $(\tilde{J}_s, \tilde{S}_2, \bar{S}_1, \tilde{T}_s), (S_1, \tilde{J}), (G^+, \bar{S}), (S, G^-), (\tilde{T}, \tilde{\tilde{S}}_2)$, related to the original one as

$$\begin{aligned}\tilde{J}_s &= 4J_s - 6J_w, & \tilde{J} &= -J_s + 3J_w, \\ \tilde{T}_s &= T - J'_s, & \tilde{T} &= 2T_s - 2J'_s, \\ \tilde{S}_2 &= S_2 + \frac{3}{c}S_1J_s - \frac{3}{c}S_1J_w - S'_1, \\ \tilde{\tilde{S}}_2 &= \bar{S}_2 - \frac{1}{c}\bar{S}_1J_s - \frac{3}{c}\bar{S}_1J_w + \bar{S}'_1.\end{aligned}\tag{3.2}$$

All the newly defined currents, except for \tilde{J} , are primary with respect to the Virasoro stress-tensor \tilde{T}_s (it corresponds to the choice of $b = 0, g = -1$ in eq. (3.1) and Table 2) and have the following spins and statistics: $(1^B, \frac{3}{2}^F, \frac{3}{2}^F, 2^B), (\frac{1}{2}^F, 1^B), (\frac{1}{2}^B, 1^F), (2^F, \frac{5}{2}^B), (2^B, \frac{5}{2}^F)$. The spin 1 current \tilde{J} is not even quasiprimary because its OPE with \tilde{T} contains a central term.

It can be checked that the currents $(\tilde{J}_s, \tilde{S}_2, \bar{S}_1, \tilde{T}_s)$ form $N = 2$ SCA, while the remaining eight currents $\tilde{J}, G^+, G^-, S, \bar{S}, S_1, \tilde{\tilde{S}}_2, \tilde{T}$ constitute a reducible $N = 2$ supermultiplet. Namely, the sets of the currents (S_1, \tilde{J}) and (G^+, \bar{S}) form two anti-chiral $N = 2$ spin 1/2 supermultiplets, respectively fermionic and bosonic ones, with the standard linear transformation properties under $N = 2$ SUSY (the transformation law of fermionic current S_1 contains in addition a shift by the transformation parameter). However, transformation properties of the pairs (S, G^-) and $(\tilde{T}, \tilde{\tilde{S}}_2)$ are more complicated: supersymmetry \tilde{S} mixes them with the composite bosonic and fermionic currents (B_1, B_2) and (F_1, F_2) which have the spin content $(5/2, 3), (3, 5/2)$ and are defined by

$$\begin{aligned}B_1 &= \frac{1}{c}S_1S, & B_2 &= \frac{1}{c}(\tilde{J}\tilde{T} - G^+G^- - 2S_1\tilde{\tilde{S}}_2 - S\bar{S}), \\ F_1 &= \frac{1}{c}(G^-S_1 - S\tilde{J}), & F_2 &= \frac{2}{c}(S_1\tilde{T} - G^+S).\end{aligned}\tag{3.3}$$

These transformation properties follow from the OPEs:

$$\begin{aligned}\tilde{S}_2(z_1)S(z_2) &= \frac{2B_1}{z_{12}}, \\ \tilde{S}_2(z_1)G^-(z_2) &= \frac{2S}{z_{12}^2} + \frac{2F_1 + S'}{z_{12}}, \\ \tilde{S}_2(z_1)\tilde{T}(z_2) &= \frac{F_2}{z_{12}}, \\ \tilde{S}_2(z_1)\tilde{\tilde{S}}_2(z_2) &= \frac{\tilde{T}}{z_{12}^2} - \frac{B_2 - \frac{1}{2}\tilde{T}}{z_{12}}.\end{aligned}\tag{3.4}$$

So the eight currents

$$(S_1, \tilde{J}), (G^+, \bar{S}), (S, G^-), (\tilde{T}, \tilde{\tilde{S}}_2)$$

together form a nonlinear and actually not fully reducible representation of the $N = 2$ SCA defined above.

Crucial for putting this representation in a more transparent manifestly supersymmetric form is the observation that the nonlinearly transforming pairs of the basic currents, namely (S, G^-) and $(\tilde{T}, \tilde{\tilde{S}}_2)$, can be combined with the composites B_1, F_1 and F_2, B_2 into two *linearly* transforming spin 2 $N = 2$ supermultiplets with the opposite overall Grassmann parities.

Thus, the basic currents of $N = 2$ super- $W_3^{(2)}$ together with the above composites split into the five irreducible linear multiplets of the $N = 2$ superconformal subalgebra

$$\text{bosonic, spin 1} : (\tilde{J}_s, \tilde{\tilde{S}}_2, \bar{S}_1, \tilde{T}_s) \quad (N = 2 \text{ SCA}) , \quad (3.5)$$

$$\text{fermionic, spin } \frac{1}{2} : (S_1, \tilde{J}) ,$$

$$\text{bosonic, spin } \frac{1}{2} : (G^+, \bar{S}) , \quad (3.6)$$

$$\text{fermionic, spin 2} : (S, G^-, B_1, F_1) ,$$

$$\text{bosonic, spin 2} : (\tilde{T}, \tilde{\tilde{S}}_2, F_2, B_2) . \quad (3.7)$$

This extended set of currents, in accordance with their spin content, is naturally accommodated by the five $N = 2$ supercurrents: general spin 1 $J(Z)$, spin 1/2 anti-chiral fermionic $G(Z)$ and bosonic $Q(Z)$, spin 2 fermionic $F(Z)$ and bosonic $T(Z)$ ³. The precise relation of the components of these superfields to the currents of $N = 2$ $W_3^{(2)}$ is quoted in Appendix. Below we reformulate $N = 2$ $W_3^{(2)}$ in terms of SOPEs of these supercurrents.

The superfield $J(Z)$ generates the $N = 2$ SCA with SOPE

$$J(Z_1)J(Z_2) = \frac{-2c + \theta_{12}\bar{\theta}_{12}J}{Z_{12}^2} + \frac{\bar{\theta}_{12}\bar{\mathcal{D}}J - \theta_{12}\mathcal{D}J + \theta_{12}\bar{\theta}_{12}J'}{Z_{12}} , \quad (3.8)$$

where

$$\theta_{12} = \theta_1 - \theta_2 \quad , \quad \bar{\theta}_{12} = \bar{\theta}_1 - \bar{\theta}_2 \quad , \quad Z_{12} = z_1 - z_2 + \frac{1}{2}(\theta_1\bar{\theta}_2 - \theta_2\bar{\theta}_1) \quad , \quad (3.9)$$

and $\mathcal{D}, \bar{\mathcal{D}}$ are the spinor covariant derivatives defined by

$$\mathcal{D} = \frac{\partial}{\partial\theta} - \frac{1}{2}\bar{\theta}\frac{\partial}{\partial z} \quad , \quad \bar{\mathcal{D}} = \frac{\partial}{\partial\bar{\theta}} - \frac{1}{2}\theta\frac{\partial}{\partial z} , \quad (3.10)$$

$$\{\mathcal{D}, \bar{\mathcal{D}}\} = -\frac{\partial}{\partial z} \quad , \quad \{\mathcal{D}, \mathcal{D}\} = \{\bar{\mathcal{D}}, \bar{\mathcal{D}}\} = 0.$$

The next SOPEs express the property that the remaining four supecurrents have the aforementioned spins with respect to this $N = 2$ SCA

$$J(Z_1)G(Z_2) = \frac{-c\theta_{12} + \frac{1}{2}\theta_{12}\bar{\theta}_{12}G}{Z_{12}^2} + \frac{\theta_{12}\bar{\theta}_{12}G' - \theta_{12}\mathcal{D}G - G}{Z_{12}} , \quad (3.11)$$

³By Z we denote the coordinates of 1D $N = 2$ superspace, $Z = (z, \theta, \bar{\theta})$.

$$\begin{aligned}
J(Z_1)Q(Z_2) &= \frac{\frac{1}{2}\theta_{12}\bar{\theta}_{12}Q}{Z_{12}^2} + \frac{\theta_{12}\bar{\theta}_{12}Q' - \theta_{12}\mathcal{D}Q - Q}{Z_{12}} , \\
J(Z_1)F(Z_2) &= \frac{2\theta_{12}\bar{\theta}_{12}F}{Z_{12}^2} + \frac{\theta_{12}\bar{\theta}_{12}F' + \bar{\theta}_{12}\overline{\mathcal{D}}F - \theta_{12}\mathcal{D}F}{Z_{12}} , \\
J(Z_1)T(Z_2) &= \frac{2\theta_{12}\bar{\theta}_{12}T}{Z_{12}^2} + \frac{\theta_{12}\bar{\theta}_{12}T' + \bar{\theta}_{12}\overline{\mathcal{D}}T - \theta_{12}\mathcal{D}T}{Z_{12}} .
\end{aligned} \tag{3.12}$$

Let us pay attention to the presence of a central term in (3.11). It reflects the property that the superfield $G(Z)$ transforms inhomogeneously under $N = 2$ SCA. All other superfields are primary with respect to the $N = 2$ SCA supercurrent $J(Z)$.

In each of the supercurrents $F(Z)$ and $T(Z)$, the spin 3 component and one of the spin 5/2 components are composite (see (3.3)). In the superfield language, this implies that these superfields have to satisfy some constraints. Using the formulas (A.2) of Appendix, one can check that the relations (3.3) amount to the following nonlinear constraints

$$A_1 \equiv \overline{\mathcal{D}}F + \frac{2}{c}(GF) = 0 , \tag{3.13}$$

$$A_2 \equiv \overline{\mathcal{D}}T + \frac{2}{c}(GT) + \frac{2}{c}(QF) = 0 . \tag{3.14}$$

For completeness, we add also the chirality conditions for G, Q

$$\overline{\mathcal{D}}G = 0 , \quad \overline{\mathcal{D}}Q = 0 . \tag{3.15}$$

By means of eq. (3.14) one could, in principle, eliminate $F(Z)$ in terms of $T(Z), G(Z)$ and $Q(Z)$. If one substitutes this expression for $F(Z)$ in the constraint (3.13), the latter is satisfied identically. However, this expression is singular at $Q(Z) = 0$. We prefer to deal with two constrained supercurrents in order to have polynomial non-singular expressions in all SOPEs.

Now we are ready to construct the remaining SOPEs of $N = 2$ super- $W_3^{(2)}$. Taking the most general Ansatz for these SOPEs in terms of the introduced superfields, using (3.13), (3.14), (3.3) and requiring the latter to be consistent with the OPEs for the superfield components (see Appendix) we obtain the following non-trivial relations

$$\begin{aligned}
G(Z_1)Q(Z_2) &= -\frac{\frac{1}{2}\theta_{12}Q}{Z_{12}} , \\
G(Z_1)F(Z_2) &= \frac{\frac{1}{2}\theta_{12}F}{Z_{12}} , \\
G(Z_1)T(Z_2) &= -2\frac{c\theta_{12} + \theta_{12}\bar{\theta}_{12}G}{Z_{12}^3} - \frac{\theta_{12}\bar{\theta}_{12}(G' - \frac{2}{c}G\mathcal{D}G - \frac{1}{c}JG - \frac{1}{2}\overline{\mathcal{D}}J)}{Z_{12}^2} \\
&\quad + \frac{\theta_{12}(J + 2\mathcal{D}G) - 2G}{Z_{12}^2} + \frac{\overline{\mathcal{D}}J - 2G' + \frac{4}{c}G\mathcal{D}G + \frac{2}{c}JG}{Z_{12}} , \\
Q(Z_1)F(Z_2) &= 2\frac{c\theta_{12} + \theta_{12}\bar{\theta}_{12}G}{Z_{12}^3} + \frac{2G - \theta_{12}(J + 2\mathcal{D}G) + \theta_{12}\bar{\theta}_{12}(G' - \frac{2}{c}G\mathcal{D}G - \frac{1}{c}JG - \frac{1}{2}\overline{\mathcal{D}}J)}{Z_{12}^2} \\
&\quad + \frac{2G' - \frac{4}{c}G\mathcal{D}G - \frac{2}{c}JG - \overline{\mathcal{D}}J + \frac{1}{2}\theta_{12}T}{Z_{12}} ,
\end{aligned}$$

$$\begin{aligned}
Q(Z_1)T(Z_2) &= -2\frac{\theta_{12}\bar{\theta}_{12}Q}{Z_{12}^3} + \frac{2\theta_{12}\mathcal{D}Q - 2Q - \theta_{12}\bar{\theta}_{12}(Q' - \frac{2}{c}\mathcal{D}GQ - \frac{1}{c}JQ - \frac{2}{c}G\mathcal{D}Q)}{Z_{12}^2} \\
&\quad + \frac{\frac{4}{c}\mathcal{D}GQ + \frac{4}{c}G\mathcal{D}Q + \frac{2}{c}JQ - 2Q'}{Z_{12}}, \\
F(Z_1)T(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}(\frac{2}{c}JF - 3F' + \frac{12}{c}\mathcal{D}GF) + 2\theta_{12}\mathcal{D}F + 2\bar{\theta}_{12}\bar{\mathcal{D}}F + 4F}{Z_{12}^2} \\
&\quad + \frac{\theta_{12}\bar{\theta}_{12}(-\frac{1}{2}F'' - \frac{3}{2}[\mathcal{D}, \bar{\mathcal{D}}]F' + \frac{2}{c}JF' - \frac{3}{c}\mathcal{D}J\bar{\mathcal{D}}F - \frac{2}{c}\bar{\mathcal{D}}J\mathcal{D}F + \frac{2}{c}\mathcal{D}G'F)}{Z_{12}} \\
&\quad + \frac{\theta_{12}\bar{\theta}_{12}(\frac{2}{c}\mathcal{D}GF' + \frac{6}{c}G'\mathcal{D}F + \frac{6}{c}G\mathcal{D}F' - \frac{2}{c^2}\mathcal{D}JGF)}{Z_{12}} \\
&\quad + \frac{\theta_{12}(2\mathcal{D}F' + \frac{2}{c}\mathcal{D}JF - \frac{2}{c}J\mathcal{D}F - \frac{4}{c}\mathcal{D}F\mathcal{D}G) - \bar{\theta}_{12}(\frac{2}{c}\bar{\mathcal{D}}JF - \frac{1}{c}J\bar{\mathcal{D}}F - \frac{3}{c}\bar{\mathcal{D}}F')}{Z_{12}} \\
&\quad + \frac{\bar{\theta}_{12}(\frac{2}{c}G'F + \frac{6}{c}GF' + \frac{3}{c}\mathcal{D}G\bar{\mathcal{D}}F - \frac{2}{c^2}JGF - \frac{2}{c^2}(\mathcal{D}GGF)) + 2F'}{Z_{12}}, \\
T(Z_1)T(Z_2) &= \frac{4T + 2\theta_{12}\mathcal{D}T + 2\bar{\theta}_{12}\bar{\mathcal{D}}T + \theta_{12}\bar{\theta}_{12}(\frac{2}{c}JT - \frac{12}{c}F\mathcal{D}Q + \frac{12}{c}\mathcal{D}GT - 3T')}{Z_{12}^2} \\
&\quad + \frac{\bar{\theta}_{12}(2\bar{\mathcal{D}}T' + \frac{4}{c}QF' + \frac{8}{c^2}GF\mathcal{D}Q + \frac{4}{c}GT' + \frac{2}{c}J\bar{\mathcal{D}}T + \frac{4}{c}\mathcal{D}G\bar{\mathcal{D}}T - \frac{2}{c}T\bar{\mathcal{D}}J)}{Z_{12}} \\
&\quad + \frac{2T' + \theta_{12}(2\mathcal{D}T' + \frac{4}{c}\mathcal{D}F\mathcal{D}Q - \frac{2}{c}J\mathcal{D}T - \frac{4}{c}\mathcal{D}G\mathcal{D}T + \frac{2}{c}T\mathcal{D}J)}{Z_{12}} \\
&\quad - \frac{\theta_{12}\bar{\theta}_{12}(T'' + [\mathcal{D}, \bar{\mathcal{D}}]T' - \frac{2}{c}\mathcal{D}T\bar{\mathcal{D}}J + \frac{4}{c}\mathcal{D}TG' + \frac{4}{c}\mathcal{D}FQ' - \frac{8}{c^2}G\mathcal{D}F\mathcal{D}Q)}{Z_{12}} \\
&\quad - \frac{\theta_{12}\bar{\theta}_{12}(\frac{8}{c^2}F\mathcal{D}G\mathcal{D}Q - \frac{2}{c}JT' + \frac{4}{c}\mathcal{D}J\bar{\mathcal{D}}T + \frac{2}{c}[\mathcal{D}, \bar{\mathcal{D}}]F\mathcal{D}Q + \frac{4}{c}\mathcal{D}T'G)}{Z_{12}} \\
&\quad - \frac{\theta_{12}\bar{\theta}_{12}(\frac{4}{c}\mathcal{D}F'Q + \frac{2}{c}F'\mathcal{D}Q - \frac{4}{c^2}QF\mathcal{D}J - \frac{4}{c^2}GT\mathcal{D}J)}{Z_{12}} \\
&\quad - \frac{\theta_{12}\bar{\theta}_{12}(\frac{4}{c}F\mathcal{D}Q' - \frac{4}{c}\mathcal{D}GT' - \frac{4}{c}\mathcal{D}G'T)}{Z_{12}}. \tag{3.16}
\end{aligned}$$

The above SOPEs are self-consistent only on the shell of constraints (3.13), (3.14). These constraints are first class and the Jacobi identities are satisfied only on their shell $A_1 = A_2 = 0$. They are consistent with the SOPEs (3.8), (3.11), (3.12), (3.16) in the sense that the SOPEs of A_1 , A_2 with all supercurrents are vanishing on the constraint shell (the compatibility of the whole set of SOPEs with the linear chirality conditions (3.15) is evident by construction). It should be pointed out that it is impossible to satisfy the Jacobi identities off the constraint shell unless one further enlarges the set of supercurrents. We have checked this by inserting the expressions A_1 and A_2 (3.13), (3.14) ⁴ in all appropriate places in the right hand sides of the SOPEs obtained. Thus the constraints (3.13), (3.14) are absolutely necessary for the above set of $N = 2$ superfields to form a closed algebra. In a forthcoming paper devoted to $N = 2$ superfield hamiltonian reduction [15] it will be shown that these

⁴ A_1 and A_2 are non-zero off the constraints shell.

constraints (as well as the chirality conditions (3.15)) are remnants of the Hull-Spence type constraints [18] for the supercurrents of $N = 2$ extension of affine superalgebra $sl(3|2)^{(1)}$.

Our final remark concerns the presence of the spin 1/2 currents S_1 and G^+ in the basis (3.2). At first sight, following the reasonings of ref. [19], one could think that they can be factored out to yield a smaller nonlinear algebra. However, this is not true in the present case because an important assumption of ref. [19] does not hold, namely the assumption that OPEs between the spin 1/2 currents contain singularities. Indeed, the OPEs of these currents are regular in our case. So in the algebra $N = 2$ super- $W_3^{(2)}$ the spin 1/2 currents cannot be removed.

4 Superfield reduction to $N = 2$ super- W_3 algebra

In this section we show that, if one imposes the additional first-class constraint on the supercurrent $Q(Z)$

$$\tilde{Q}(Z) \equiv Q(Z) - c = 0 \quad (4.1)$$

and the condition

$$G(Z) = 0 \quad (4.2)$$

which fixes a gauge with respect to gauge transformations generated by (4.1) (together (4.1) and (4.2) form a set of second-class constraints), one arrives at SOPEs of some self-consistent nonlinear algebra written in terms of unconstrained supercurrents. This algebra turns out to be none other than the well-known $N = 2$ super- W_3 algebra [12] formulated in terms of $N = 2$ superfields in [13].

Let us define new superfields $\tilde{J}(Z)$ and $\tilde{T}(Z)$

$$\begin{aligned} \tilde{J}(Z) &= J(Z) - 2\mathcal{D}G(Z) + 2\partial Q(Z), \\ \tilde{T}(Z) &= T(Z) - \frac{1}{3}[\mathcal{D}, \overline{\mathcal{D}}]J(Z) + \frac{4}{3}\mathcal{D}\partial G(Z) - \frac{4}{9c}J(Z)^2 - \frac{20}{9c}(\partial J(Z)Q(Z)) + \\ &\quad \frac{2}{9c}J(Z)\partial Q(Z) - \frac{2}{c}\overline{\mathcal{D}}J(Z)\mathcal{D}Q(Z) - \frac{16}{9c}\mathcal{D}G(Z)\mathcal{D}G(Z) + \frac{20}{9c}\partial Q(Z)\partial Q(Z) + \\ &\quad \partial J(Z) - \frac{2}{3}\partial^2 Q(Z). \end{aligned} \quad (4.3)$$

This substitution is dictated by the requirement that SOPEs of these supercurrents with $G(Z)$ and $\tilde{Q}(Z)$ (4.1) be homogeneous in $\tilde{Q}(Z)$ and $G(Z)$. The supercurrent $\tilde{J}(Z)$ can be checked to generate another $N = 2$ SCA, such that the conformal weights of $\tilde{Q}(Z)$, $G(Z)$, $T(Z)$ and $F(Z)$ with respect to it equal 0, 1/2, 2 and 5/2, respectively. The constraints (4.1) and (4.2) prove to be preserved by this $N = 2$ SCA.

Thus the superfields $\tilde{J}(Z)$ and $\tilde{T}(Z)$ by construction are gauge invariant with respect to the gauge transformations generated by the first-class constraints (4.1). So, according to the standard ideology of hamiltonian reduction [16, 3, 17]⁵, they have to form a closed superalgebra (with all the Jacobi identities satisfied) on the shell of constraints (4.1), (4.2)

⁵Actually, the described procedure supplies a nice example of secondary hamiltonian reduction in $N = 2$ superspace [15].

and with

$$F = -\frac{1}{2}\overline{\mathcal{D}}T. \quad (4.4)$$

The last relation follows by substituting (4.1), (4.2) into eq.(3.14). Note that with this F eq.(3.13) is identically satisfied.

Using the SOPEs of $N = 2$ super- $W_3^{(2)}$ we find that the resulting SOPEs for the currents $\tilde{J}(Z)$ and $\tilde{T}(Z)$ after substituting (4.1), (4.2), (4.4) exactly coincide with SOPEs of classical $N = 2$ super- W_3 algebra [13]. In the next Section we will make use of this result to construct the simplest nontrivial hamiltonian flow on $N = 2$ super- $W_3^{(2)}$.

5 Generalized $N = 2$ super Boussinesq equation

The most general hamiltonian which can be constructed out of the five superfields $\tilde{J}(Z)$, $G(Z)$, $\tilde{Q}(Z)$, $\tilde{T}(Z)$ and $F(Z)$ of $N = 2$ super- $W_3^{(2)}$ algebra under the natural assumptions that it (i) respects rigid $N = 2$ supersymmetry and (ii) has the same scaling dimension 2 as the hamiltonian of the ordinary bosonic Boussinesq equation, is given by

$$\mathcal{H} = \int dzd\theta d\bar{\theta} \left(T + v_1 J^2 + v_2 J \mathcal{D}G + v_3 J \partial Q + v_4 \mathcal{D}G \partial Q \right). \quad (5.1)$$

Note the presence of the free parameters v_1, \dots, v_4 in (5.1). Now, using SOPEs of $N = 2$ super- $W_3^{(2)}$ algebra and the definition

$$\frac{\partial \phi}{\partial t} = \{ \phi, \mathcal{H} \} \quad (5.2)$$

(here $\phi(z)$ is any supercurrent of $N = 2$ super- $W_3^{(2)}$ and the Poisson brackets in the r.h.s. of (5.2) are understood), it is straightforward to find the explicit form of the evolution equations. Due to the complexity of these equations, it is not so illuminating to write down them here. We also postpone to future publications the analysis of integrability of this system.

In ref. [13] we have constructed, in $N = 2$ superfield form, the most general one-parameter super Boussinesq equation with the second hamiltonian structure given by the classical $N = 2$ super- W_3 algebra. With making use of the results of Sect. 4 it is not difficult to show that the obtained system of evolution equations reproduces the one of ref. [13] upon the above truncation of $N = 2$ super- $W_3^{(2)}$ to $N = 2$ super- W_3 and with the following relations between the parameters in (5.1)

$$v_1 = \frac{4\alpha - \frac{8}{3c}}{6}, \quad v_2 = -\frac{4}{c} - 4v_1 \quad v_3 = 4v_1, \quad (5.3)$$

Here α is the parameter entering the $N = 2$ super Boussinesq equation [13].

6 Conclusion

To summarize, we have concisely rewritten classical $N = 2$ super- $W_3^{(2)}$ algebra of ref. [8] in terms of five constrained $N = 2$ superfields, found its superfield reduction to $N = 2$

super- W_3 algebra [12, 13], and constructed a family of $N = 2$ supersymmetric equations the second hamiltonian structure for which is given by this superalgebra and which generalize the $N = 2$ super Boussinesq equation of ref. [13]. In a forthcoming publication [20] we will extend our consideration to the case of full quantum $N = 2$ super- $W_3^{(2)}$ algebra.

An interesting problem is to find out possible string theory implications of $N = 2$ $W_3^{(2)}$ algebra, both in its component and superfield formulations. The fact that there exists a zero central charge stress-tensor (2.9) with respect to which almost all of the currents are primary suggests that this algebra admits an interpretation as a kind of twisted topological superconformal algebra and so has a natural realization in terms of BRST structure associated with some string (the $W_3^{(2)}$ one?) or superstring.

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Appendix

Here we present the component OPEs for the $N = 2$ super- $W_3^{(2)}$ algebra [8] and give the relation between the currents of the latter and the components of $N = 2$ supercurrents $J(Z)$, $G(Z)$, $Q(Z)$, $F(Z)$ and $T(Z)$.

The whole set of the OPEs contains, besides those of the subalgebras $W_3^{(2)}$ and $N = 2$ SCA (2.7), (2.8), the following non-trivial relations:

$$\begin{aligned}
J_w(z_1)S_1(z_2) &= -\frac{\frac{1}{6}S_1}{z_{12}}, \quad J_s(z_1)S_1(z_2) = -\frac{\frac{1}{2}S_1}{z_{12}}, \quad J_s(z_1)J_w(z_2) = \frac{\frac{c}{3}}{z_{12}^2}, \\
J_s(z_1)T_w(z_2) &= \frac{2J_w}{z_{12}^2}, \quad J_s(z_1)G^+(z_2) = -\frac{G^+}{z_{12}}, \quad J_w(z_1)S(z_2) = \frac{\frac{1}{3}S}{z_{12}}, \\
J_s(z_1)S_2(z_2) &= -\frac{\frac{1}{2}S_2}{z_{12}}, \quad J_w(z_1)T_s(z_2) = \frac{\frac{2}{3}J_s}{z_{12}^2}, \quad J_w(z_1)S_2(z_2) = -\frac{\frac{1}{6}S_2}{z_{12}}, \\
T_s(z_1)T_w(z_2) &= \frac{\frac{4}{c}(S_1\bar{S}_1 + J_wJ_s)}{z_{12}^2} + \frac{\frac{2}{c}(S_1\bar{S}_2 + S_1\bar{S}'_1 - S_2\bar{S}_1 + S'_1\bar{S}_1 + 2J_wJ'_s)}{z_{12}}, \\
T_s(z_1)G^+(z_2) &= -\frac{\frac{2}{c}(G^+J_s - S_1\bar{S})}{z_{12}}, \quad T_s(z_1)S_1(z_2) = -\frac{\frac{1}{2}S_1}{z_{12}^2} + \frac{S_2 - S'_1 - \frac{1}{c}(S_1J_s + 3J_wS_1)}{2z_{12}}, \\
T_s(z_1)S_2(z_2) &= -\frac{3S_1}{z_{12}^3} + \frac{2S_2 - 3S'_1 + \frac{1}{c}(3S_1J_s - 9J_wS_1)}{2z_{12}^2} + \frac{\frac{2}{c}G^+S - \frac{4}{c}S_1T_s + \frac{3}{c^2}S_1J_s^2 + \frac{1}{c}S_1J'_s}{2z_{12}} \\
&\quad - \frac{\frac{3}{c}S_2J_s - \frac{3}{c}J_wS_2 - \frac{6}{c^2}J_wJ_sS_1 + \frac{9}{c^2}J_w^2S_1 + \frac{6}{c}J_wS'_1 - \frac{2}{c}S'_1J_s + \frac{3}{c}J'_wS_1 - S'_2 + S''_1}{2z_{12}},
\end{aligned}$$

$$\begin{aligned}
T_w(z_1)S_1(z_2) &= -\frac{\frac{1}{2}S_1}{z_{12}^2} - \frac{S_2 - \frac{1}{c}S_1J_s + \frac{5}{c}J_wS_1 + S_1'}{2z_{12}}, & T_w(z_1)S(z_2) &= -\frac{\frac{2}{c}G^-S_1 - \frac{2}{c}J_wS}{z_{12}}, \\
T_w(z_1)S_2(z_2) &= \frac{3S_1}{z_{12}^3} + \frac{2S_2 - \frac{3}{c}S_1J_s + \frac{9}{c}J_wS_1 + 3S_1'}{2z_{12}^2} - \frac{\frac{2}{c}G^+S + \frac{4}{c}S_1T_w - \frac{1}{c^2}S_1J_s^2 + \frac{1}{c}S_1J_s'}{2z_{12}} \\
&\quad - \frac{\frac{1}{c}S_2J_s - \frac{1}{c}J_wS_2 + \frac{6}{c^2}J_wJ_sS_1 - \frac{21}{c^2}J_w^2S_1 - \frac{6}{c}J_wS_1' + \frac{2}{c}S_1'J_s - \frac{3}{c}J_w'S_1 - S_2' - S_1''}{2z_{12}}, \\
G^+(z_1)S(z_2) &= -\frac{2S_1}{z_{12}^2} - \frac{S_2 - \frac{1}{c}S_1J_s - \frac{3}{c}J_wS_1 + S_1'}{z_{12}}, & G^+(z_1)S_2(z_2) &= -\frac{\frac{3}{2c}G^+S_1}{z_{12}}, \\
G^-(z_1)S_1(z_2) &= -\frac{\frac{1}{2}S}{z_{12}}, & G^-(z_1)S_2(z_2) &= \frac{\frac{3}{2}S}{z_{12}^2} - \frac{\frac{1}{c}G^-S_1 + \frac{1}{c}J_sS - \frac{9}{c}J_wS - S'}{2z_{12}}, \\
S_1(z_1)\bar{S}(z_2) &= \frac{G^+}{2z_{12}}, & S_1(z_1)\bar{S}_2(z_2) &= \frac{T_s + T_w + \frac{2}{c}S_1\bar{S}_1 - \frac{1}{c}J_s^2 - \frac{3}{c}J_w^2}{2z_{12}}, \\
S(z_1)S_2(z_2) &= \frac{3S_1S}{2cz_{12}}, & S(z_1)\bar{S}_2(z_2) &= -\frac{3G^-}{2z_{12}^2} - \frac{\frac{3}{c}G^-J_s + \frac{1}{c}\bar{S}_1S - \frac{3}{c}J_wG^- + G^{-'}}{2z_{12}}, \\
S_2(z_1)S_2(z_2) &= \frac{2S_1S_2}{cz_{12}}, & S_1(z_1)\bar{S}_1(z_2) &= -\frac{\frac{c}{2}}{z_{12}^2} + \frac{\frac{3}{2}J_w - \frac{1}{2}J_s}{z_{12}}, \\
S_2(z_1)\bar{S}_2(z_2) &= \frac{3c}{z_{12}^4} + \frac{3J_s - 9J_w}{z_{12}^3} + \frac{2T_s - 2T_w + \frac{1}{c}J_s^2 - \frac{18}{c}J_wJ_s + \frac{33}{c}J_w^2 + 3J_s' - 9J_w'}{2z_{12}^2} \\
&\quad + \frac{\frac{1}{c}G^+G^- + \frac{1}{c}S_1\bar{S}_2 + \frac{1}{c}S\bar{S} + \frac{1}{c}S_2\bar{S}_1 - \frac{1}{2c^2}J_s^3 + \frac{1}{c}T_sJ_s - \frac{1}{c}T_wJ_s - \frac{3}{c}J_wT_s + \frac{3}{c}J_wT_w - \frac{3}{2c^2}J_wJ_s^2}{z_{12}} \\
&\quad + \frac{\frac{33}{2c^2}J_w^2J_s - \frac{45}{2c^2}J_w^3 - \frac{9}{2c}J_wJ_s' + \frac{1}{2c}J_s'J_s - \frac{9}{2c}J_w'J_s + \frac{33}{2c}J_w'J_w + \frac{1}{2}T_s' - \frac{1}{2}T_w' + \frac{1}{2}J_s'' - \frac{3}{2}J_w''}{z_{12}}. \quad (A.1)
\end{aligned}$$

Here we omitted the OPEs which can be obtained from (A.1) via the discrete automorphisms: $J_{w,s} \rightarrow -J_{w,s}$, $G^\pm \rightarrow \pm G^\mp$, $S \rightarrow \bar{S}$, $\bar{S} \rightarrow S$, $S_1 \rightarrow \bar{S}_1$, $\bar{S}_1 \rightarrow -S_1$, $S_2 \rightarrow -\bar{S}_2$, $\bar{S}_2 \rightarrow S_2$.

The currents are related to the components of the $N = 2$ $W_3^{(2)}$ supercurrents in the following way

$$\begin{aligned}
J| &= \tilde{J}_s, \quad \bar{\mathcal{D}}J| = \sqrt{2}\tilde{S}_2, \quad \mathcal{D}J| = \sqrt{2}\bar{S}_1, \quad [\mathcal{D}, \bar{\mathcal{D}}]J| = -2\tilde{T}_s, \\
G| &= -\sqrt{2}S_1, \quad \mathcal{D}G| = \tilde{J}, \quad Q| = \sqrt{2}G^+, \quad \mathcal{D}Q| = \bar{S}, \\
F| &= S, \quad \mathcal{D}F| = -\frac{1}{\sqrt{2}}G^-, \quad \bar{\mathcal{D}}F| = 2\sqrt{2}B_1, \quad \mathcal{D}\bar{\mathcal{D}}F| = 2F_1, \\
T| &= \tilde{T}, \quad \mathcal{D}T| = -\sqrt{2}\tilde{S}_2, \quad \bar{\mathcal{D}}T| = \sqrt{2}F_2, \quad \mathcal{D}\bar{\mathcal{D}}T| = -2B_2. \quad (A.2)
\end{aligned}$$

The composite currents B_1, B_2, F_1, F_2 were defined in (3.3).

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